Information-Theoretic Study on Routing Path Selection in Two-Way Relay Networks

Shanshan Wu, Wenguang Mao, and Xudong Wang
UM-SJTU Joint Institute, Shanghai Jiao Tong University, Shanghai, China
Email: {wushanshan, maowenguang1987, wxudong}@sjtu.edu.cn

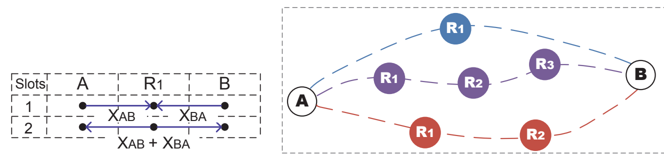
***Abstract*—Two-way relaying is a promising technique to improve network throughput. However, how to efficiently apply it to a wireless network remains an unresolved issue. In this paper, we consider routing path selection in a two-way relay network. Information theoretical analysis is carried out to derive bandwidth efficiency and energy efficiency of a linear multi-hop network with transmission through two-way relay channels. Such analysis provides a framework of routing path selection by considering bandwidth efficiency, energy efficiency, and latency subject to physical layer constraints such as the transmission rate, processing power, path length, and the number of relays. This framework provides insightful guidelines for future routing protocol design in a two-way relay network.**

I. Introduction

A typical model for a two-way relay channel (TWRC) contains three nodes [1]–[3]: two source nodes $\mathbb{A}$ and $\mathbb{B}$ exchange data via a relay $\mathbb{R}_1$ (see Fig. 1(a)). Assuming half-duplex transmission, then an amplify-and-forward (AF) TWRC [3] works as follows: in the first time slot, $\mathbb{A}$ and $\mathbb{B}$ transmit simultaneously to $\mathbb{R}_1$, where the superimposed signals get amplified and transmitted in the second time slot. After receiving it, $\mathbb{A}$ and $\mathbb{B}$ subtract their own signal to extract the desired data. By exploiting bi-directional interference, TWRC enables two source nodes to exchange data every two time slots, which is half the time needed in a traditional routing scheme. In spite of the advantage of TWRC in a three-node scenario, how to efficiently utilize it to acquire performance gains in a wireless network remains an unsolved problem.

The objective of this paper is to select an optimal routing path when transmitting data through TWRCs (Fig. 1(b)), by choosing energy efficiency (EE), bandwidth efficiency (BE), and latency as performance metrics. To achieve this goal, we first develop a framework to evaluate performance of a linear multi-hop network. This framework is then applied to routing path selection in a two-way relay network, where an optimal path is selected by jointly considering EE, BE, and latency.

The contribution of this paper is twofold. First, an information theoretical framework is developed in Section III to derive the EE, BE, and latency of a linear multi-hop network, assuming that Hop-by-Hop scheduling scheme is used to enable transmission through TWRCs. Besides, an optimal power allocation scheme is derived, which allows a multi-hop network to consume the smallest energy with a given transmission rate. Second, in Section IV we numerically evaluate the EE-BE performance of a linear multi-hop network as a function of different variables, such as the transmission rate and total number of relays. Specifically, we find that: 1) when transmission rate is low, network with fewer relays has higher EE and lower latency; 2) when transmission rate is high, network with more relays consumes less energy at the cost of higher latency. Since high EE, high BE, and low latency cannot be achieved simultaneously, we define a general objective function to integrate the three performance metrics. Based on this function, an optimal route to provide the best tradeoff between EE, BE, and latency can be determined.



(a) (b)

Fig. 1. (a) Illustration of an AF TWRC; (b) $\mathbb{A}$ and $\mathbb{B}$ want to exchange data through AF TWRCs, which one of the three routes performs the best?

Although there are different two-way relaying techniques (e.g., decode-and-forward), this paper is focused on AF TWRC because of its simplicity in implementation and comparable performance with other techniques [1] [3].

II. System Model

We consider a linear multi-hop network with two source nodes $\mathbb{A}$ and $\mathbb{B}$, and k relays $\mathbb{R}_1$, ..., $\mathbb{R}_k$ in between. $\mathbb{A}$ and $\mathbb{B}$ want to exchange packets with each other, with the help of k relays over AF TWRCs. Every node is assumed to have single antenna and one-hop transmission range, and operate in half-duplex mode. Nodes within two hops cannot transmit simultaneously, except in TWRCs.

To enable efficient transmissions over AF TWRCs, a simple scheduling scheme is proposed in [4], which allows $\mathbb{A}$ and $\mathbb{B}$ to exchange one packet every four time slots[1]. To see how this

The research work is supported by National Natural Science Foundation of China (NSFC) No. 61172066, the Ministry of Education (MOE) Program for New Century Excellent Talents, and Oriental Scholar Program of Shanghai Municipal Education Commission. The authors would like to thank these sponsors for their generous support.

[1]A different multi-hop scheme is proposed in [3], which allows $\mathbb{A}$ and $\mathbb{B}$ to exchange one packet every two time slots. However, it is not applicable in real systems, because noise will accumulate rapidly as packets traverse the network, resulting in a fast decreasing SNR for new transmitted packets. Therefore, we do not consider the scheme in [3].

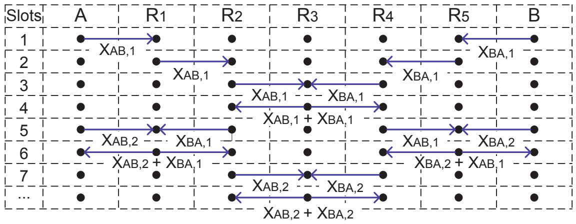


Fig. 2. Illustration of the Hop-by-Hop scheme for $k = 5$.

scheme works, let us take $k = 5$ as an example (see Fig. 2). Let $X_{\mathbb{AB},n}$ ($X_{\mathbb{BA},n}$) be the n-th packets that $\mathbb{A}$ ($\mathbb{B}$) wants to send to $\mathbb{B}$ ($\mathbb{A}$). During the first two time slots, $X_{\mathbb{AB},1}$ and $X_{\mathbb{BA},1}$ are inserted into the network and are forwarded to relays $\mathbb{R}_2$ and $\mathbb{R}_4$, respectively. An AF TWRC is then formed among the three relays in the center, allowing $\mathbb{R}_2$ and $\mathbb{R}_4$ to exchange their packets. In the fifth and sixth time slots, two TWRCs are formed symmetrically on both sides of the network, where $\mathbb{R}_2$ ($\mathbb{R}_4$) swaps its packet with $\mathbb{A}$ ($\mathbb{B}$), so that $X_{\mathbb{BA},1}$ and $X_{\mathbb{AB},1}$ reach their destinations, and two new packets $X_{\mathbb{AB},2}$ and $X_{\mathbb{BA},2}$ enter the system. By then, packets are transmitted in a stable and recursive pattern: during every four time slots, the source nodes will insert one new pair of packets into the system, and receive one pair of packets from the other side; all the relays in between help forward data over AF TWRCs.

The scheduling scheme proposed in [4] only considers networks with *odd* number of relays. In the next section we generalize this scheme to take into account all values of k, and name it a Hop-by-Hop scheme because TWRCs are formed every two hops. As illustrated in Fig. 4–8, the recursive pattern varies for different values of k: when k is odd, all the nodes are involved in TWRCs; when k is even, one of the source nodes will perform traditional transmission.

III. Performance Measures

In this section we investigate the performance of linear multi-hop networks with a Hop-by-Hop scheduling scheme. The performance metrics of interest are: EE (bits/J), BE (bits/s/Hz), and latency (time slots/bit). EE and BE are measures of the efficiency that a network utilizes energy and spectrum to transmit data [5] [6], respectively, while latency refers to the time elapse for each bit to reach its destination after being sent out. By deriving those metrics, we are then motivated to determine the highest EE when BE is given. Since EE and BE depends on how power is provisioned at each node, an optimal power allocation scheme associated with the highest EE value will also be determined.

Since Shannon's capacity formula is applied to derive the optimal power allocation, all sources of interference are taken into account in the SINR part. Therefore, the complexity of derivation increases as the number of relays (i.e., k) grows. In this paper, we only consider small-scale multi-hop networks with $0 \leq k \leq 6$. However, our framework can be easily extended to the case when $k > 7$. Moreover, as shown in Section IV, analyzing 7 cases (i.e., $0 \leq k \leq 6$) is enough to reveal the performance trend of linear multi-hop networks.

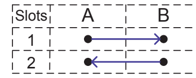


Fig. 3. Illustration of the Hop-by-Hop scheme: $k = 0$.

For each value of k and each node i, let x_i and y_i denote the transmitted and received symbols, respectively. z_i represents a zero-mean AWGN with power spectral density N_0. Let P_i be the transmission energy[2] carried by each transmitted symbol x_i. Let P_{proc} denote the processing energy. According to [7],

$$P_{proc} = (1/\eta - 1)P_{tr} + P_0, \tag{1}$$

where η is a constant representing the drain efficiency of the power amplifier at the transmitter. The first term of Eq. (1) denotes processing energy in power amplifier, which is linearly proportional to the transmission energy P_{tr}. The second term P_0 denotes energy consumed in radio electronics (at both transmitter and receiver sides) other than power amplifier, and is assumed to be a constant value. Since P_{proc} depends on the network scenario, in the following derivation, we will use $P_{proc,k}$ and $P_{0,k}$ to indicate networks with k relays.

For simplicity, we assume that all the nodes are equally spaced and transmit with the same rate R (bits/channel use[3]). The channel[4] between two consecutive nodes is denoted as h. The relationship[5] between h and distance d is $|h|^2 = d^{-\alpha}$, where α is pass-loss exponent and normally $2 \leq \alpha \leq 4$.

A. k=0

As shown in Fig. 3, $\mathbb{A}$ communicates with $\mathbb{B}$ directly.

1) BE and EE: Define $U = P_{\mathbb{A}} + P_{\mathbb{B}}$, then it denotes the transmission energy during two channel uses. According to Eq. (1), the corresponding processing energy in the two channel uses is: $P_{proc,0} = (1/\eta - 1)U + P_{0,0}$. Then the total energy is: $U + P_{proc,0} = U/\eta + P_{0,0}$. Hence,

$$\text{BE} = R, \quad \text{EE} = 2R/(U/\eta + P_{0,0}). \tag{2}$$

2) Optimal Power Allocation: Given BE, i.e., given R, EE is maximized when U reaches its minimum, which is achieved when we use capacity-achieving coding, i.e.,

$$R = \log_2(1 + |h|^2 P_i/N_0), \ i = \mathbb{A}, \ \mathbb{B}. \tag{3}$$

Accordingly, the minimum U and the optimal powers are

$$U_{\min} = 2N_0|h|^{-2}(2^R - 1), \quad P_{\mathbb{A}} = P_{\mathbb{B}} = U_{\min}/2. \tag{4}$$

B. k=1

This case is depicted in Fig. 1(a), where $\mathbb{A}$, $\mathbb{R}_1$, and $\mathbb{B}$ form a TWRC. Packets are exchanged every 2 time slots.

[2] P_i is the transmission power (unit: J/s) evaluated in a time scale of one symbol, so it has a unit of J/symbol or J/channel use.

[3] Each channel use occupies 1 second $\times$ hertz.

[4] Here channel is assumed to be reciprocal.

[5] This is a common assumption on the relationship between channel gain and distance, e.g., [5].

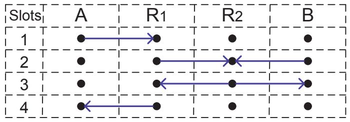


Fig. 4. Illustration of the Hop-by-Hop scheme: $k = 2$.

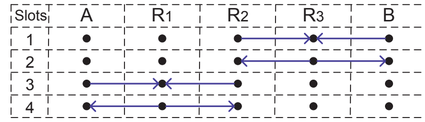


Fig. 5. Illustration of the Hop-by-Hop scheme: $k = 3$.

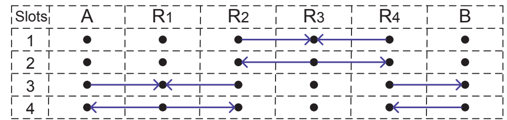


Fig. 6. Illustration of the Hop-by-Hop scheme: $k = 4$.

1) BE and EE: Define the transmission energy during the two channel uses as $T = P_{\mathbb{A}} + P_{\mathbb{R}_1} + P_{\mathbb{B}}$, then following the same procedure as in previous case, we get the total energy: $T/\eta + P_{0,1}$. Hence,

$$\text{BE} = R, \;\; \text{EE} = 2R/(T/\eta + P_{0,1}). \tag{5}$$

2) Optimal Power Allocation: Given BE, the maximum EE is achieved when T is minimum. As shown in Fig. 1(a), $\mathbb{A}$, $\mathbb{B}$, and $\mathbb{R}_1$ forms a TWRC. Assuming that the signal received at $\mathbb{R}_1$ is amplified by β, i.e.,

$$\beta = \sqrt{P_{\mathbb{R}_1}/(|h|^2 P_{\mathbb{A}} + |h|^2 P_{\mathbb{B}} + N_0)}, \tag{6}$$

after self-interference cancellation, signals at $\mathbb{A}$ and $\mathbb{B}$ become

$$\begin{aligned} y_{\mathbb{A}} - h^2\beta x_{\mathbb{A}} &= h^2\beta x_{\mathbb{B}} + h\beta z_{\mathbb{R}_1} + z_{\mathbb{A}}, \\ y_{\mathbb{B}} - h^2\beta x_{\mathbb{B}} &= h^2\beta x_{\mathbb{A}} + h\beta z_{\mathbb{R}_1} + z_{\mathbb{B}}. \end{aligned} \tag{7}$$

Energy is used with maximum efficiency when transmission is done by capacity-achieving coding, i.e.[6],

$$R = \log_2(1 + \frac{|h|^4\beta^2 P_{\mathbb{B}}}{(|h|^2\beta^2+1)N_0}) = \log_2(1 + \frac{|h|^4\beta^2 P_{\mathbb{A}}}{(|h|^2\beta^2+1)N_0}). \tag{8}$$

Rearrange Eq. (8), substitute it into T, take the derivative of T with respect to β, and let it be zero gives

$$\beta^2 = \sqrt{(2^{R+1}-2)|h|^{-4}/(2^{R+1}-1)}. \tag{9}$$

Checking the second derivation of T shows that when β satisfies Eq. (9), T reaches its mimimum

$$T_{\min} = 2N_0|h|^{-2}(\sqrt{(2^{R+1}-1)(2^{R+1}-2)} + 2^{R+1} - 2), \tag{10}$$

which gives a maximum EE. The corresponding optimal power allocation at $\mathbb{A}$, $\mathbb{B}$, and $\mathbb{R}_1$ can be found by substituting Eq. (9) into Eqs. (8) and (6). Their expressions can be found in our report [8] and are omitted here for saving space[7].

C. k=2

The recursive pattern for $k = 2$ is shown in Fig. 4. During the first and fourth time slots, $\mathbb{A}$ sends and receives one packet from $\mathbb{R}_1$ through unicast transmission. During the second and third time slots, $\mathbb{R}_1$, $\mathbb{R}_2$, and $\mathbb{B}$ form a TWRC. Packets between $\mathbb{A}$ and $\mathbb{B}$ are exchanged every 4 time slots.

1) BE and EE: Let $U = P_{\mathbb{A}} + P_{\mathbb{R}_1}$ and $T = P_{\mathbb{R}_1} + P_{\mathbb{R}_2} + P_{\mathbb{B}}$ be the transmission energy dissipated in unicast channel and TWRC, respectively[8], following the same procedure as in the case of $k = 0$ gives the total energy: $(U+T)/\eta + P_{0,2}$. Hence,

$$\text{BE} = R/2, \;\; \text{EE} = 2R/((U+T)/\eta + P_{0,2}). \tag{11}$$

2) Optimal Power Allocation: Given BE, EE is maximized when $U + T$ is minimized. Since energy in U and T are consumed in different time slots, $U + T$ achieves minimum when both of U and T are minimized. On the other hand, the minimum of U and T, and the associated optimal power allocation scheme have been found in the cases of $k = 0$ and $k = 1$, so previous results, e.g., Eqs. (4) and (10), can be directly applied here.

D. k=3

The case of $k = 3$ is shown in Fig. 5, where the left half nodes and the right half nodes form two TWRCs, respectively. $\mathbb{A}$ and $\mathbb{B}$ exchange one packet every 4 time slots.

1) BE and EE: Let $T_1 = P_{\mathbb{A}} + P_{\mathbb{R}_1} + P_{\mathbb{R}_2}$ and $T_2 = P_{\mathbb{R}_2} + P_{\mathbb{R}_3} + P_{\mathbb{B}}$ represent the transmission energy dissipated in the two TWRCs, following the same procedure as in the case of $k = 0$ gives us the total energy: $(T_1 + T_2)/\eta + P_{0,3}$. Hence,

$$\text{BE} = R/2, \;\; \text{EE} = 2R/((T_1+T_2)/\eta + P_{0,3}). \tag{12}$$

2) Optimal Power Allocation: Given BE, EE is maximized when $T_1 + T_2$ is minimized. For the same reason as in the case of $k = 2$, previous results, e.g., Eq. (10), can be directly applied here to get the minimum of $T_1 + T_2$ and the associated optimal power allocation scheme.

E. k=4

This case is shown in Fig. 6. During the first two time slots, data is exchanged between $\mathbb{R}_2$ and $\mathbb{R}_4$. During the last two time slots, each of $\mathbb{A}$ and $\mathbb{B}$ sends and receives a new packet. Accordingly, a recursive pattern of 4 time slots is formed.

[6]Here β is assumed to be a real number. If it is a complex number, then we need to use $|\beta|$ instead of β during derivation.

[7]Note that there are slight differences between the symbols defined in this paper and those defined in our report, i.e., [8].

[8]Note that U and T have already been defined in the previous cases, but they are reused here to denote energy consumption in the same transmission scenario, i.e., U for unicast channel and T for TWRC. For the same reason, symbols like S and F are reused in the following derivation.

1) BE and EE: Let $T = P_{\mathbb{R}_2} + P_{\mathbb{R}_3} + P_{\mathbb{R}_4}$, $F = P_{\mathbb{A}} + P_{\mathbb{R}_1} + P_{\mathbb{R}_2} + P_{\mathbb{R}_4} + P_{\mathbb{B}}$ denote the transmission energy consumed during the first two and last two time slots, respectively. Following the same procedure as in the case of $k = 0$, we get the total energy consumption: $(T + F)/\eta + P_{0,4}$. Hence,

$$\text{BE} = R/2, \ \ \text{EE} = 2R/((T + F)/\eta + P_{0,4}). \tag{13}$$

2) Optimal Power Allocation: Given BE, EE is maximized when $T + F$ is minimized. Since T and F represents energy consumed in different time slots, $T + F$ is minimized when both of T and F are minimized. $T_{\min}$ have been derived in the case of $k = 1$, i.e., Eq. (10), so we only need to find $F_{\min}$.

During the third and fourth time slots, $\mathbb{A}$ communicates with $\mathbb{R}_2$ through a TWRC. At the same time, $\mathbb{B}$ and $\mathbb{R}_4$ perform unicast transmission. Therefore, we have

$$y_{\mathbb{R}_1} = hx_{\mathbb{A}} + hx_{\mathbb{R}_2} + z_{\mathbb{R}_1} + \sqrt{3^{-\alpha}}hx_{\mathbb{R}_4}, \tag{14}$$

$$x_{\mathbb{R}_1} = \beta y_{\mathbb{R}_1}, \tag{15}$$

$$y_{\mathbb{A}} = hx_{\mathbb{R}_1} + z_{\mathbb{A}} + \sqrt{5^{-\alpha}}hx_{\mathbb{B}}, \tag{16}$$

$$y_{\mathbb{R}_2} = hx_{\mathbb{R}_1} + z_{\mathbb{R}_2} + \sqrt{3^{-\alpha}}hx_{\mathbb{B}}, \tag{17}$$

$$y_{\mathbb{B}} = hx_{\mathbb{R}_4} + z_{\mathbb{B}} + \sqrt{3^{-\alpha}}hx_{\mathbb{R}_2} + \sqrt{5^{-\alpha}}hx_{\mathbb{A}}, \tag{18}$$

$$y_{\mathbb{R}_4} = hx_{\mathbb{B}} + z_{\mathbb{R}_4} + \sqrt{3^{-\alpha}}hx_{\mathbb{R}_1}, \tag{19}$$

where $\sqrt{3^{-\alpha}}$ and $\sqrt{5^{-\alpha}}$ come from two assumptions: 1) relays are equally spaced along a line, and 2) relationship between the channel h and distance d is $|h|^2 = d^{-\alpha}$.

EE achieves maximum when capacity-achieving coding is applied, therefore, the following SINR values at receivers $\mathbb{A}$, $\mathbb{R}_2$, $\mathbb{B}$, and $\mathbb{R}_4$, respectively, should all be equal to $2^R - 1$:

$$\frac{|h|^4\beta^2 P_{\mathbb{R}_2}}{3^{-\alpha}|h|^4\beta^2 P_{\mathbb{R}_4} + (|h|^2\beta^2 + 1)N_0 + 5^{-\alpha}|h|^2 P_{\mathbb{B}}}, \tag{20}$$

$$\frac{|h|^4\beta^2 P_{\mathbb{A}}}{3^{-\alpha}|h|^4\beta^2 P_{\mathbb{R}_4} + (|h|^2\beta^2 + 1)N_0 + 3^{-\alpha}|h|^2 P_{\mathbb{B}}}, \tag{21}$$

$$\frac{|h|^2 P_{\mathbb{R}_4}}{3^{-\alpha}|h|^2 P_{\mathbb{R}_2} + 5^{-\alpha}|h|^2 P_{\mathbb{A}} + N_0}, \tag{22}$$

$$\frac{|h|^2 P_{\mathbb{B}}}{3^{-\alpha}|h|^2 P_{\mathbb{R}_1} + N_0}. \tag{23}$$

Additionally, Eqs. (15) and (14) give

$$P_{\mathbb{R}_1} = \beta^2(|h|^2 P_{\mathbb{A}} + |h|^2 P_{\mathbb{R}_2} + 3^{-\alpha}|h|^2 P_{\mathbb{R}_4} + N_0). \tag{24}$$

There are five equations, i.e., (20)–(24), and five variables, i.e., $P_{\mathbb{A}}$, $P_{\mathbb{R}_1}$, $P_{\mathbb{R}_2}$, $P_{\mathbb{R}_4}$, $P_{\mathbb{B}}$. Solving these equations and substituting the results into the formula of F gives[9]

$$F = (c_1\beta^2 + \frac{c_2}{|h|^4\beta^2} + c_3|h|^{-2})N_0 \geq (2\sqrt{c_1c_2} + c_3)N_0|h|^{-2}, \tag{25}$$

where $\{c_i\}_{i=1}^3$ are complicated functions of R and α. Accordingly, the minimum value of F is $F_{\min} = (2\sqrt{c_1c_2} + c_3)N_0|h|^{-2}$, achieved when $\beta^4 = c_2|h|^{-4}/c_1$. The corresponding optimal power allocation can be obtained by solving Eqs. (20)–(24) and substituting the optimal β.

The complete expressions of $F_{\min}$, and the optimal power, as well as the detailed derivation are available in [8], which are omitted here for saving space[7].

[9]Here we do not prove that $c_1c_2 > 0$ and $c_4c_5 > 0$, but $F_{\min}$ and $S_{\min}$ are achievable as demonstrated in Section IV.

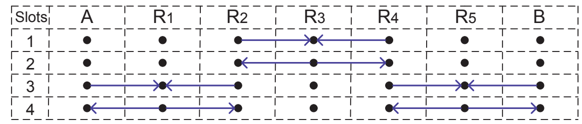


Fig. 7. Illustration of the Hop-by-Hop scheme: $k = 5$.

F. k=5

The recursive pattern for $k = 5$ is shown in Fig. 7, which has been analyzed in Section II.

1) BE and EE: Define $T = P_{\mathbb{R}_2} + P_{\mathbb{R}_3} + P_{\mathbb{R}_4}$ and $S = P_A + P_{\mathbb{R}_1} + P_{\mathbb{R}_2} + P_{\mathbb{R}_4} + P_{\mathbb{R}_5} + P_B$ as the transmission energy consumed during the first and last two time slots, respectively. Following the same procedure as in the case of $k = 0$ gives the total energy consumption: $(T + S)/\eta + P_{0,5}$. Hence,

$$\text{BE} = R/2, \ \ \text{EE} = 2R/((T + S)/\eta + P_{0,5}). \tag{26}$$

2) Optimal Power Allocation: Similar to the previous case, given BE, EE is maximized when both T and S are minimized. Since $T_{\min}$ is given in Eq. (10), we only need to find $S_{\min}$ and the corresponding power.

During the third and fourth time slots, two TWRCs are formed. Due to symmetry, we only need to consider one of them, and the total power is minimized when $P_{\mathbb{A}} = P_{\mathbb{B}}$, $P_{\mathbb{R}_2} = P_{\mathbb{R}_4}$, $P_{\mathbb{R}_1} = P_{\mathbb{R}_5}$. Accordingly, $S = 2(P_{\mathbb{A}} + P_{\mathbb{R}_1} + P_{\mathbb{R}_2})$. Considering the left TWRC formed by $\mathbb{A}$, $\mathbb{R}_1$, and $\mathbb{R}_2$ gives

$$y_{\mathbb{R}_1} = hx_{\mathbb{A}} + hx_{\mathbb{R}_2} + \sqrt{3^{-\alpha}}hx_{\mathbb{R}_4} + \sqrt{5^{-\alpha}}hx_{\mathbb{B}} + z_{\mathbb{R}_1}, \tag{27}$$

$$x_{\mathbb{R}_1} = \beta y_{\mathbb{R}_1}, \tag{28}$$

$$x_{\mathbb{A}} = hx_{\mathbb{R}_1} + \sqrt{5^{-\alpha}}hx_{\mathbb{R}_5} + z_{\mathbb{A}}, \tag{29}$$

$$x_{\mathbb{R}_2} = hx_{\mathbb{R}_1} + \sqrt{3^{-\alpha}}hx_{\mathbb{R}_5} + z_{\mathbb{R}_2}, \tag{30}$$

where $\sqrt{3^{-\alpha}}$ and $\sqrt{5^{-\alpha}}$ follow the same reason as in the previous case.

Energy is minimized when capacity-achieving coding is used, so the following SINR values at receivers $\mathbb{A}$ and $\mathbb{R}_2$ should be equal to $2^R - 1$:

$$\frac{|h|^4\beta^2 P_{\mathbb{R}_2}}{|h|^2(P_{\mathbb{R}_1} - \beta^2|h|^2(P_{\mathbb{A}} + P_{\mathbb{R}_2})) + 5^{-\alpha}|h|^2 P_{\mathbb{R}_5} + N_0}, \tag{31}$$

$$\frac{|h|^4\beta^2 P_{\mathbb{A}}}{|h|^2(P_{\mathbb{R}_1} - \beta^2|h|^2(P_{\mathbb{A}} + P_{\mathbb{R}_2})) + 3^{-\alpha}|h|^2 P_{\mathbb{R}_5} + N_0}. \tag{32}$$

From Eqs. (27) and (28), we have

$$P_{\mathbb{R}_1} = \beta^2|h|^2(P_{\mathbb{A}} + P_{\mathbb{R}_2} + 3^{-\alpha}P_{\mathbb{R}_4} + 5^{-\alpha}P_{\mathbb{B}} + N_0|h|^{-2}). \tag{33}$$

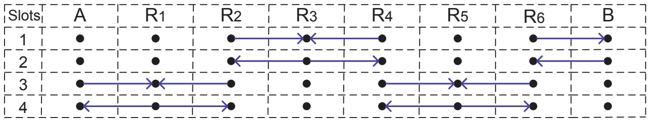


Fig. 8. Illustration of the Hop-by-Hop scheme: $k = 6$.

Similar to the case of $k = 4$, we can solve the 3 equations, i.e., (31)–(33) to get $P_{\mathbb{A}}$, $P_{\mathbb{R}_1}$, and $P_{\mathbb{R}_2}$, and then substitute them into the formula of S to get[9]

$$S = (c_4\beta^2 + \frac{c_5}{|h|^4\beta^2} + c_6|h|^{-2})N_0 \geq (2\sqrt{c_4c_5} + c_6)N_0|h|^{-2}, \quad (34)$$

where $\{c_i\}_{i=4}^{6}$ are complicated functions of R and α. Accordingly, $S_{\min} = (2\sqrt{c_4c_5} + c_6)N_0|h|^{-2}$, achieved when $\beta^4 = c_5|h|^{-4}/c_4$. The corresponding optimal power can be obtained by substituting the optimal β in Eqs. (31)–(33).

The complete expressions and the detailed derivation are available in [8], which are omitted here for saving space[7].

G. k=6

As shown in Fig. 8, the system works like the case of $k = 4$ during the first two time slots, and the case of $k = 5$ during the last two time slots.

1) BE and EE: Define $F = P_{\mathbb{R}_2} + P_{\mathbb{R}_3} + P_{\mathbb{R}_4} + P_{\mathbb{R}_6} + P_{\mathbb{B}}$ and $S = P_{\mathbb{A}} + P_{\mathbb{R}_1} + P_{\mathbb{R}_2} + P_{\mathbb{R}_4} + P_{\mathbb{R}_5} + P_{\mathbb{R}_6}$ as the transmission energy during the first and the last two time slots, respectively. Following the same procedure as in the case of $k = 0$ gives the total energy consumption: $(F + S)/\eta + P_{0,6}$. Hence,

$$\text{BE} = R/2, \;\; \text{EE} = 2R/((F + S)/\eta + P_{0,6}). \quad (35)$$

2) Optimal Power Allocation: Similar to the previous case, given BE, EE is maximized when both F and S are minimized. Because of the same recursive patterns, $F_{\min}$ and $S_{\min}$ as well as the optimal power allocation have been derived in previous cases of $k = 4$ and $k = 5$, e.g., in Eqs. (25) and (34).

H. Latency

As shown in Fig. 4–8, packets are forwarded to the next hop per time slot, so the latency of a multi-hop network with k relays is $k + 1$ time slots/bit.

IV. Numerical Analysis

In previous section, we have derived EE, BE, and latency of multi-hop networks with different number of relays. We also found the condition when EE is maximized for a given BE, i,e, when the network consumes the smallest energy for a given end-to-end rate. The goal of this section is to numerically investigate those results. Note that the EE discussed in this section always refers to the *maximum* EE for a given BE.

The path loss exponent α is 4 and noise power spectral density N_0 is -174 dBm/Hz. To capture the effect of processing energy, we consider two cases: an extreme case of zero processing energy, i.e., $\eta = 1$ and $P_{0,k} = 0$ for all k; and a specific case with[10] $\eta = 0.75$. For the second case, we assume that $P_{0,k}$ is proportional to the number of senders and receivers in a recursive pattern, i.e., $P_{0,k}/P_{0,0} = m/4$, where m is the number of active nodes in a recursive pattern for network with k relays. Let $P_{0,0} = 5 \times 10^{-7}$ mJ/channel use, then $P_{0,1} = 6/4P_{0,0}$, $P_{0,2} = 10/4P_{0,0}$, $P_{0,3} = 12/4P_{0,0}$, $P_{0,4} = 16/4P_{0,0}$, $P_{0,5} = 18/4P_{0,0}$, $P_{0,6} = 22/4P_{0,0}$.

In Fig. 9(a), we compare the EE-BE performance for $k = 0$ and $k = 1$, which corresponds to transmission directly between $\mathbb{A}$ and $\mathbb{B}$ or through a TWRC. The processing energy is ignored. Unlike direct transmission, where EE and BE are involved in a tradeoff relation, transmission through a TWRC presents a different EE-BE interaction: EE first increases and then decreases with BE, achieving maximum when BE is around 0.6 bits/s/Hz. In other words, at a low transmission rate, we can decrease energy consumption and increase transmission rate at the same time. Besides, when transmission rate approaches zero, direct transmission tends to have higher EE than TWRC, and hence performs better.

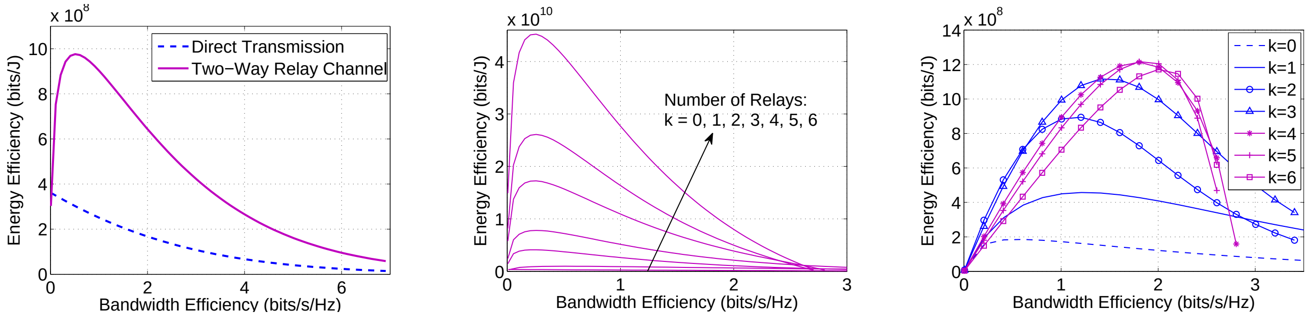
In Fig. 9(b), we plot EE and BE of a linear multi-hop network with a fixed distance between $\mathbb{A}$ and $\mathbb{B}$, but different number of relays. The processing energy is ignored. Given the length of a multi-hop network and a certain BE, EE increases with increased number of relays. This is because when the length of a multi-hop network is fixed, more relays means smaller distance in each hop, and hence smaller transmission energy consumed in each hop. Although more relays increases the number of devices consuming energy, the decrease due to a smaller distance per hop dominates the total energy.

In Fig. 9(c), we depict EE and BE for the previous case, but with processing energy considered. As shown in Fig. 9(c), when transmission rate is low, routes with fewer relays have higher EE. This is because at low rate, transmission energy is small, processing energy dominates the total energy consumption. Since $P_{0,k}$ is assumed to be proportional to the number of active nodes in a recursive pattern, routes with fewer relays dissipate less processing energy and hence have larger EE. As rate increases, the EE-BE relation resembles that in Fig. 9(b), i.e., a network with more relays tends to have larger EE. When BE approaches 2.8 bits/channel use, a sharp decrease occurs for EE of networks with more than 3 relays. This is because an upper bound exists for the transmission rate when $k \leq 3$, due to the interference. Taking $k = 4$ as an example, since $P_{\mathbb{R}_2}$ in the numerator, and $P_{\mathbb{R}_4}$, $P_{\mathbb{B}}$ in the denominator are of the same order, R is sure to have an upper bound.

To apply our results to the selection of an optimal routing path in Fig. 1(b), we assume that: 1) relays along each route are equidistant; 2) each route has a small curvature so that it can be treated as an elongated straight line[11]. Previous analysis tells us that high EE, high BE, and low latency cannot be achieved simultaneously. In order to find an optimal routing path that gives the best tradeoff between those performance

[10]This drain efficiency is achievable for high-class power amplifiers [7].

[11]Routes satisfying the two assumptions have better performance. A TWRC with relay in the middle has the highest sum rate and lowest outage probability [9]. Besides, transmission through a path with large curvature means that a large amount of energy is spent on moving data around the source node rather than forwarding it to the destination, so it is energy inefficient.



(a) (b) (c)

Fig. 9. Comparisons of the EE-BE relation for a multi-hop network with end-to-end distance 1000 m: (a) $k = 0$ versus $k = 1$ with zero processing energy; (b) $0 \leq k \leq 6$ with zero processing energy; (c) $0 \leq k \leq 6$ with processing energy considered.

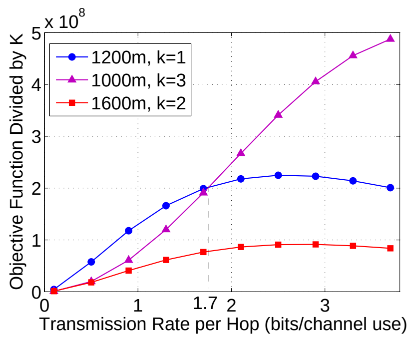


Fig. 10. Comparisons of the three routes in Fig. 1(b), with processing energy considered.

metrics, we define a general performance metric as[12]

$$M = \frac{\mathrm{EE}/\mathrm{EE}_{\max} \times \mathrm{BE}/\mathrm{BE}_{\max}}{\mathrm{latency}/\mathrm{latency}_{\max}} = \frac{\mathrm{EE} \times \mathrm{BE}}{\mathrm{latency}} \times K, \qquad (36)$$

where $\mathrm{EE}_{\max}$, $\mathrm{BE}_{\max}$, and $\mathrm{latency}_{\max}$ are the maximum achievable EE, BE, and latency of a system. Therefore, K is a constant formed by the three maximum values. M is an objective function that is to be maximized.

In Fig. 10, we depict M/K for the three routes in Fig. 1(b), assuming that their lengths and the number of relays are 1200 m with $k = 1$, 1000 m with $k = 3$, and 1600 m with $k = 2$. When the transmission rate is smaller than 1.7 bits/channel use, the first route is optimal, mainly because it has the lowest latency and smallest processing energy consumption. When the transmission rate is large, the second route is optimal, which is consistent with our previous analysis: since the second route has both the shortest length and the largest number of relays among the three routes, it consumes the smallest transmission energy, and hence has the largest M value. This example illustrates the necessity of jointly considering the transmission rate, processing energy, path length, and the number of relays, in order to select an optimal path that provides the best tradeoff between EE, BE, and latency.

[12]Note that the definition of this general performance metric is not unique. For example, if one cares more about EE when selecting the routing path, one can design an objective function that gives EE more "weight".

V. Conclusion

In this paper, an information theoretical framework was developed to study routing path selection for amplify-and-forward two-way relay networks. EE, BE, and latency for linear multi-hop networks were formulated under a Hop-by-Hop scheduling scheme. An optimal power allocation scheme was also derived. It allows minimum energy consumption for a given transmission rate.

To select an optimal path that provides the best tradeoff between EE, BE, and latency, a joint consideration of the transmission rate, processing energy, path length, and the number of relays is necessary. At a low transmission rate, where processing energy dominates the total energy consumption, route with fewer relays has higher energy efficiency and lower latency. As the rate increases, transmission energy becomes dominant. Route with a shorter length and more relays tends to consume less energy at the cost of higher latency.

Our research work was limited to routing paths with equally spaced relays and small curvature. Overcoming these limitations is subject to future research.

References

[1] P. Popovski and H. Yomo, "The anti-packets can increase the achievable throughput of a wireless multi-hop network," in *Proc. IEEE ICC*, June 2006.

[2] S. Zhang, S. C. Liew, and P. P. Lam, "Hot topic: physical-layer network coding," in *Proc. ACM MobiCom*, pp. 358–365, Sept. 2006.

[3] P. Popovski and H. Yomo, "Bi-directional amplification of throughput in a wireless multi-hop network," in *Proc. IEEE VTC 2006-Spring*, pp. 588–593, May 2006.

[4] Q. You, Z. Chen, Y. Li, and B. Vucetic, "Multi-hop bi-directional relay transmission schemes using amplify-and-forward and analog network coding," in *Proc. IEEE ICC*, June 2011.

[5] C. Bae, and W. E. Stark, "End-to-end energy-bandwidth tradeoff in multihop wireless networks," *IEEE Trans. Inf. Theory*, vol. 55, no. 9, pp. 4051–4066, Sept. 2009.

[6] S. Verdú, "Spectral efficiency in the wideband regime," *IEEE Trans. Inf. Theory*, vol. 48, no. 6, pp. 1319–1343, June 2002.

[7] Q. Wang, M. Hempstead, and W. Yang, "A realistic power consumption model for wireless sensor network devices," in *Proc. IEEE SECON*, pp. 286–295, Sept. 2006.

[8] S. Wu and X. Wang. (2012, July 27). *Information-Theoretic Study on Routing Path Selection in Two-Way Relay Networks* [Online]. Available: http://arxiv.org/abs/1207.6650

[9] Z. Yi, M. Ju, and I. M. Kim, "Outage probability and optimum power allocation for analog network coding," *IEEE Trans. Wireless Commun.*, vol. 10, no. 2, pp. 407–412, Feb. 2011.